Example 1: Solve the following LPP by using simplex method;

$$Max\ \ Z = 6x_1 + 4x_2$$

Subject to

$$x_1 + 2x_2 \leq 720$$
$$2x_1 + x_2 \leq 780$$
$$x_1 \leq 320$$

Solution:

Step 1: Convert the Following LPP into Standard Form

$$Max\ Z = 6x_1 + 4x_2 + 0S_1 + 0S_2 + 0S_3$$

Subject to

$x_1 + 2x_2 + S_1 =$

$720\ 2x_1 + x_2 + S_2$

$= 780$

$x_1 + S_3 = 320$

Step 2: Initial Basic Feasible Solution

$x_1 = 0$ *and* $x_2 = 0$ in the above equation then we have $S_1 = 720$, $S_2 = 780$ *and* $S_3 = 320$

Table 1: Initial Solution

		$C_j \longrightarrow$	6	4	0	0	0	
C_B	B	$b(= x_B)$	x_1	x_2	S_1	S_2	S_3	Min.Ratio
0	S_1	720	1	2	1	0	0	$\frac{720}{1} = 720$
0	S_2	780	2	1	0	1	0	$\frac{780}{2} = 390$
0	S_3	320	1	0	0	0	1	$\frac{320}{1} = 320 \rightarrow$
$Z = 0$		$Z_j =$	0	0	0	0	0	
		$C_j - Z_j$	6 $\uparrow$	4	0	0	0	

Step 3: Perform the Optimality Test

Since all $C_j - Z_j \geq 0(j = 1, 2)$, the current solution is not optimal. Variable x_1 is chosen to enter into the basis as $C_1 - Z_1 = 6$ is the largest positive number in the x_1 column, where all elements are positive. This means that for every unit of variable x_1, the objective function will increase in value by 6. The x_1 column is the key column.

Step 4: Determine the Variable to Leave the Basis

The variable to leave the basis is determined by dividing the value in the x_B- (constant) column by their corresponding elements in the key column as shown in Table 1. Since the exchange ratio, 320 is minimum in row 3, the basic variable S_3 is chosen to leave the solution basis.

Iteration 1: Since the key element enclosed in the circle in Table 1 is 1, this row remain unchanged. The new values of the elements in the remaining rows for the new Table is obtained by performing the following elementary row operations on all rows so that all elements except the key element 1 in the key column are zero.

$$R_3(new) \rightarrow \frac{R_3(old)}{1(keyelement)} = (320, 1, 0, 0, 0, 1)$$

$$R_2(new) \rightarrow R_2(old) - 2R_3(new)$$

$$R_2(new) \rightarrow (780, 2, 1, 0, 1, 0) - 2(320, 1, 0, 0, 0, 1) = (140, 0, 1, 0, 1, -2)$$

$$R_1(new) \rightarrow R_1(old) - 1R_3(new)$$

$$R_1(new) \rightarrow (720, 1, 2, 1, 0, 0) - 1(320, 1, 0, 0, 0, 1) = (400, 0, 2, 1, 0, -1)$$

Then, the new improved solution is given in 2 below;
An improved basic feasible solution can be read from Table 2 as: $x_1 = 320$, $S_2 = 140$, $S_3 = 400$ *and* $x_2 = 0$. The improved value of objective function is Z=1920.

Once again, calculate values of $C_j - Z_j$ in the same manner as we have done to get the improved solution in Table 2 to see whether the solution is optimal or not. Since $C_2 - Z_2 > 0$, the current solution is not optimal.

Table 4.2: Improved Solution

		$C_j \longrightarrow$	6	4	0	0	0	
C_B	B	$b(=x_B)$	x_1	x_2	S_1	S_2	S_3	Min.Ratio
0	S_1	400	0	2	1	0	-1	$\frac{400}{2} = 200$
0	S_2	140	0	1	0	1	-2	$\frac{140}{1} = 140 \rightarrow$
6	x_1	320	1	0	0	0	1	
$Z = 1920$		$Z_j =$	6	0	0	0	6	
		$C_j - Z_j$	0	4 ↑	0	0	-6	

Iteration 2: Repeats steps 3 to 4. Table 3 is obtained by performing following row operations to enter x_2 into the basis and to drive out S_2 from the basis.

$$R_2(new) \rightarrow \frac{R_2(old)}{1(key\ element)} = (140, 0, 1, 0, 1, -2)$$

$$R_1(new) \rightarrow R_1(old) - 2R_2(new)$$
$$R_1(new) \rightarrow (400,0,2,1,0,-1) - 2(140,0,1,0,1,-2) = (120,0,0,1,-2,3)$$
$$R_3(new) \rightarrow R_3(old) - 0R_2(new)$$
$$R_3(new) \rightarrow (320,1,0,0,0,1) - 0(140,0,1,0,1,-2) = (320,1,0,0,0,1)$$

Then, the improved solution for iteration 2 is given in Table 3 below;

Table 3: Improved Solution

		$C_j \longrightarrow$	6	4	0	0	0	
C_B	B	$b(=x_B)$	x_1	x_2	S_1	S_2	S_3	Min.Ratio
0	S_1	120	0	0	1	-2	3	$\frac{120}{3} = 40 \rightarrow$
4	x_2	140	0	1	0	1	-2	
6	x_1	320	1	0	0	0	1	$\frac{320}{1} = 320$
$Z = 2480$		$Z_j =$	6	4	0	4	-2	
		$C_j - Z_j$	0	0	0	-4	2 ↑	

Iteration 3: Repeats steps 3 to 4. Table 4 is obtained by performing following row operations to enter S_3 into the basis and to drive out S_1 from the basis.

$$R_1(new) \rightarrow \frac{R_1(old)}{3(key\,element)} = (40, 0, 0, 1/3, -2/3, 1)$$

$$R_2(new) \to R_2(old) + 2R_1(new)$$
$$R_2(new) \to (140,0,1,0,1,-2) + 2(40,0,0,1/3,-2/3,1) = (220,0,1,2/3,-1/3,0)$$
$$R_3(new) \to R_3(old) - 1R_1(new)$$
$$R_3(new) \to (320,1,0,0,0,1) - 1(40,0,0,1/3,-2/3,1) = (280,1,0,-1/3,2/3,0)$$

Then, the improved solution for iteration 2 is given in Table 4 below;

Table 4: Optimal Solution

		$C_j \longrightarrow$	6	4	0	0	0	
C_B	B	$b(=x_B)$	x_1	x_2	S_1	S_2	S_3	Min.Ratio
0	S_3	40	0	0	1/3	-2/3	1	
4	x_2	220	0	1	2/3	-1/3	0	
6	x_1	280	1	0	-1/3	2/3	0	
Z = 2560		Z_j =	6	4	2/3	8/3	0	
		$C_j - Z_j$	0	0	-2/3	-8/3	0	

Since all $C_j - Z_j \leq 0$ corresponding to non - basic variables columns, the current solution cannot be improved further. This means that the current basic feasible solution is also the optimal solution. Thus, x_1 = 280, x_2 = 220 and the value of objective function is Z=2560.

Example 2: Use the simplex method to solve following LP problem.

$$Max\ Z = 6x_1 + 17x_2 + 10x_3$$

Subject to

$$x_1 + x_2 + 4x_3 \leq 2000$$
$$2x_1 + x_2 + x_3 \leq 3600$$
$$x_1 + 2x_2 + 2x_3 \leq 2400$$
$$x_1 \leq 30$$

and

$$x_1, x_2, x_3 \geq 0$$

Solution:

Convert the Following LPP into Standard Form

$$Max\ Z = 6x_1 + 17x_2 + 10x_3 + 0S_1 + 0S_2 + 0S_3 + 0S_4$$

Subject to

$$x_1 + x_2 + 4x_3 + S_1 = 2000$$
$$2x_1 + x_2 + x_3 + S_2 = 3600$$
$$x_1 + 2x_2 + 2x_3 + S_3 = 2400$$
$$x_1 + S_4 = 30$$

and

$$x_1, x_2, x_3, S_1, S_2, S_3, S_4 \geq 0$$

Initial Basic Feasible Solution

An initial basic feasible solution is obtained by setting $x_1 = x_2 = x_3 = 0$. Thus, the initial solution is: $S_1 = 2000, S_2 = 3600, S_3 = 2400, S_4 = 30$ and Max Z = 0. The solution can also be read from the initial simplex Table 1

Table 1: Initial Solution

		$C_j \longrightarrow$	6	17	10	0	0	0	0	
C_B	B	$b(=x_B)$	x_1	x_2	x_3	S_1	S_2	S_3	S_4	Min.Ratio
0	S_1	2000	1	1	4	1	0	0	0	$\frac{2000}{1} = 2000$
0	S_2	3600	2	1	1	0	1	0	0	$\frac{3600}{1} = 3600$
0	S_3	2400	1	2	2	0	0	1	0	$\frac{2400}{2} = 1200 \rightarrow$
0	S_4	30	1	0	0	0	0	0	1	
$Z = 0$		$Z_j =$	0	0	0	0	0	0	0	
		$C_j - Z_j$	16	17 ↑	10	0	0	0	0	

Perform the Optimality Test

Since all $C_j - Z_j \geq 0$, the current solution is not optimal. Variable x_2 is chosen to enter into the basis as $C_2 - Z_2 = 17$ is the largest positive number in the x_2 column. We apply the following row operations to get a new improved solution and removing S_3 from the basis.

$$R_3(new) \longrightarrow \frac{R_3(old)}{2(key\ element)} = (1200, 1/2, 0, 0, 0, 1, -1/2, 0)$$

$$R_1(new) \longrightarrow R_1(old) - R_3(new) = (800, 1/2, 0, 3, 1, 0, -1/2, 0)$$

$$R_2(new) \longrightarrow R_2(old) - R_3(new) = (2400, 3/2, 0, 0, 0, 1, -1/2, 0)$$

$$R_4(new) \longrightarrow R_4(old) = (30, 1, 0, 0, 0, 0, 0, 1)$$

The new solution is shown in Table 2 below

Table 2: Improved Solution

		$C_j \longrightarrow$	6	17	10	0	0	0	0	
C_B	B	$b(=x_B)$	x_1	x_2	x_3	S_1	S_2	S_3	S_4	Min.Ratio
0	S_1	800	1/2	0	3	1	0	-1/2	0	$\frac{800}{1/2} = 1600$
0	S_2	2400	3/2	0	0	0	1	-1/2	0	$\frac{2400}{3/2} = 1600$
17	x_2	1200	1/2	1	1	0	0	1/2	0	$\frac{1200}{1/2} = 2400$
0	S_4	30	1	0	0	0	0	0	1	$\frac{30}{1} = 30 \rightarrow$
$Z = 20,000$		$Z_j =$	17/2	17	17	0	0	17/2	0	
		$C_j - Z_j$	15/2 ↑	0	-7	0	0	-17/2	0	

The solution shown in Table 2 is not optimal because $C_1 - Z_1 = 15/2$ which is positive in x_1 column. Thus, applying the following row operations to get new improved solution by entering variable x_1 into the basis and removing the variable S_4 from the basis.

$$R_4(new) \rightarrow \frac{R_4(old)}{1(key\ element)} = (30, 1, 0, 0, 0, 0, 0, 1)$$

$$R_1(new) \rightarrow R_1(old) - (1/2)R_4(new) = (785, 0, 0, 3, 1, 0, -1/2, -1/2)$$

$$R_2(new) \rightarrow R_2(old) - (3/2)R_4(new) = (2355, 0, 0, 0, 0, 1, -1/2, -3/2)$$

$$R_3(new) \rightarrow R_3(old) - (1/2)R_4(new) = (1185, 0, 1, 1, 0, 0, 1/2, -1/2)$$

Then, the improved solution for this iteration is given in Table 3 below;

Table 3: Optimal Solution

		$C_j \longrightarrow$	6	17	10	0	0	0	0
C_B	B	$b(=x_B)$	x_1	x_2	x_3	S_1	S_2	S_3	S_4
0	S_1	785	0	0	3	1	0	-1/2	-1/2
0	S_2	2355	0	0	0	0	1	-1/2	-3/2
17	x_2	1185	0	1	1	0	0	1/2	-1/2
16	x_1	30	1	0	0	0	0	0	1
Z = 20,625		Z_j =	16	17	17	0	0	17/2	15/2
		$C_j - Z_j$	0	0	-7	0	0	-17/2	-15/2

Since all $C_j - Z_j \leq 0$ corresponding to non - basic variables columns, the current solution cannot be improved further. This means that the current basic feasible solution is also the optimal solution. Thus, $x_1 = 30, x_2 = 1,185$ and $x_3 = 0$ to obtain the maximum value of Z=20,625.